A Exact Aggregation of Producer and External Surplus

This appendix records the exact relationship between the composite per-unit welfare term Ξ_i and its surplus components. Let

$$\Xi_i(s) = \theta_\eta p_i P_i(s) + \Lambda_i(s), \tag{56}$$

with $P_i(s) > 0$ and $\Lambda_i(s) > 0$. Then

$$\frac{\partial \log \Xi_i}{\partial s} = \varpi_i^V(s) \frac{\partial \log P_i}{\partial s} + [1 - \varpi_i^V(s)] \frac{\partial \log \Lambda_i}{\partial s}, \tag{57}$$

where $\varpi_i^V(s) = \theta_\eta p_i P_i(s)/\Xi_i(s)$. Differentiating again gives

$$\begin{aligned} \frac{\partial^2 \log \Xi_i}{\partial s^2} =& \varpi_i^V(s) \frac{\partial^2 \log P_i}{\partial s^2} + [1 - \varpi_i^V(s)] \frac{\partial^2 \log \Lambda_i}{\partial s^2} \\ &+ \varpi_i^V(s)[1 - \varpi_i^V(s)] \left(\frac{\partial \log P_i}{\partial s} - \frac{\partial \log \Lambda_i}{\partial s} \right)^2. \end{aligned} \tag{58}$$

Thus even if both P_i and Λ_i are log-quadratic, $\log \Xi_i$ need not be exactly log-quadratic. The closed-form model in the text should be read as a local second-order representation of the composite total per-unit social value. The general propositions do not rely on exact log-quadratic aggregation.

B Additional Proofs

B.1 Proof of the innovation and private-value optima

Using (20)-(22), log innovation output after the firm's R&D choice is

$$\log I_i^*(s) = \text{constant}_i + \frac{\eta}{2-\eta} [\log A_i(s) + \log P_i(s)] + \frac{2}{2-\eta} \log E_i(s). \tag{59}$$

Substitution yields a concave quadratic with linear coefficient $\frac{\eta}{2-\eta}(a_i + q_i)$ and curvature $\frac{\eta}{2-\eta}(\alpha_i + \pi_i) + \frac{2}{2-\eta} e_i$. Projection of the unconstrained maximizer gives (34).

Private value equals $V_i^*(s) = \frac{2-\eta}{2} p_i P_i(s) I_i^*(s)$. Therefore

$$\log V_i^*(s) = \text{constant}_i + \frac{\eta}{2-\eta} \log A_i(s) + \frac{2}{2-\eta} \log P_i(s) + \frac{2}{2-\eta} \log E_i(s). \tag{60}$$

The same quadratic argument gives (35).

C Fixed Public Costs

Suppose welfare is

$$W_i^K(s) = \Xi_i(s) I_i^*(s) - K_i(s), \tag{61}$$

where $K_i(s)$ is a fixed public cost of acquiring or maintaining state ownership. The first-order condition for an interior optimum is

$$\Xi_i(s) I_i^*(s) \Psi_i'(s) = K_i'(s), \tag{62}$$

where $\Psi_i(s) = \log[\Xi_i(s) I_i^*(s)]$. If Ψ_i is strictly concave, K_i is convex, and the left-hand side crosses the right-hand side once from above, the optimum is unique. Thus fixed public costs alter the closed form but not the economic logic: ownership is desirable only when the marginal welfare gain from innovation and strategic value exceeds the marginal public cost.

D Managerial-Effort Microfoundation for Conversion Efficiency

This appendix provides one microfoundation for the conversion-efficiency term. Let the manager allocate effort to innovation implementation e and bureaucratic activity. Suppose managerial utility is

$$U^M = w + b[1 - m(s)] e R^\eta + B m(s)(\bar{h} - e) - \frac{\psi}{2} e^2, \tag{63}$$

where $m'(s) > 0$, $b, B, \psi > 0$, and $\bar{h}$ is total effort. The interior effort choice is

$$e^*(R, s) = \frac{b[1 - m(s)] R^\eta - B m(s)}{\psi}. \tag{64}$$

Innovation is $I = e^* R^\eta$. In the interior region,

$$I(R, s) = \frac{b[1 - m(s)] R^{2\eta} - B m(s) R^\eta}{\psi}. \tag{65}$$

The cross-partial derivative is

$$\frac{\partial^2 I}{\partial R \partial s} = -\frac{\eta m'(s)}{\psi}\left[2bR^{2\eta-1} + BR^{\eta-1}\right] < 0. \tag{66}$$

Thus state ownership reduces the marginal conversion of R&D into innovation output. The log-quadratic conversion-efficiency term in the main text is a tractable reduced form for this incentive channel.

E Empirical Construction of D_i, F_i, and g_{it}

The theory distinguishes strategic dependence, foreign exposure, and geoeconomic shocks. The following constructions are examples.

1. **Strategic dependence D_i.** Use pre-shock dependence on foreign critical inputs, sector inclusion in strategic industrial plans, foreign patent-citation dependence in core technologies, or input-output exposure to bottleneck technologies.

2. **Foreign exposure F_i.** Use pre-shock export share, foreign sales, foreign ownership, foreign debt, imported intermediate share, reliance on foreign technology licenses, or share of patents filed abroad.

3. **Geoeconomic shock g_{it}.** Use shift-share exposure to tariffs, export controls, entity-list restrictions, procurement bans, investment screening, or technology-licensing restrictions, with weights measured before the shock.

A credible design should report pre-trends by D_i and F_i, allow for sector-specific trends, and avoid using post-shock outcomes to classify exposure.